A binomial experiment is a probability experiment that satisfies the following four requirements:

1. There must be a fixed number of trials

2. Each trial can have only two outcomes or outcomes that can be reduced to two outcomes. These outcomes can be considered as either success or failure

3. The outcomes of each trial must be independent of one another

4. The probability of a success must remain the same for each trial
The outcomes of a binomial experiment and the corresponding probabilities of these outcomes are called a **binomial distribution**.

**Binomial Probability Formula**

\[ P(x) = \binom{n}{x} \cdot p^x \cdot q^{n-x} \]

- \( n \): # of trials
- \( x \): # of "successes"
- \( p \): prob. of success for one trial
- \( q \): prob. of failure for one trial

\( p + q = 1 \)
\( x + n - x = n \)
Ex 1). A coin is tossed 3 times. Find the probability of getting exactly two heads.

\[ P(2H) = \binom{3}{2} \left( \frac{1}{2} \right)^2 \left( \frac{1}{2} \right)^1 \]

\[ = 3 \times \frac{1}{4} \times \frac{1}{2} = 37.5\% \]

Ex 2). A survey found that one out of five Americans say he or she has visited a doctor in any given month. If 10 people are selected at random, find the probability that exactly 3 will have visited a doctor last month.

\[ P = \text{visited doc.} \]

\[ P(3) = \binom{10}{3} \left( \frac{1}{5} \right)^3 \left( \frac{4}{5} \right)^7 \]

\[ = 20.1\% \]
Ex 3). A survey from Teenage Research Unlimited found that 30% of teenage consumers receive their spending money from part-time jobs. If 5 teenagers are selected at random, find the probability that at least 3 of them will have part-time jobs.

\[
P(\text{at least 3}) = P(3) + P(4) + P(5)
\]

\[
= \binom{5}{3} (0.30)^3 (0.70)^2 \\
+ \binom{5}{4} (0.30)^4 (0.70)^1 \\
+ \binom{5}{5} (0.30)^5 (0.70)^0
\]

\[
= 16.3\%
\]
Ex 4). Public Opinion reported that 5% of Americans are afraid of being alone in a house at night. If a random sample of 20 Americans is selected, find these probabilities by using the binomial table (Table B).

a) There are exactly 5 people in the sample who are afraid of being alone at night.

\[ n = 20 \quad P(5) = 0.002 = 0.2\% \]

b) There are at most 3 people in the sample who are afraid of being alone at night.

\[
P(\text{at most 3}) = P(0) + P(1) + P(2) + P(3) = 0.358 + 0.377 + 0.189 + 0.060 = 0.984\%
\]

c) There are at least 3 people in the sample who are afraid of being alone at night.

\[
P(\text{at least 3}) = P(3) + P(4) + \ldots + P(20) = P(\geq 3) = 1 - P(< 3) = 1 - 0.924 = 7.6\%
\]
Mean, Variance, and Standard Deviation for the Binomial Distribution

\[ \mu = np \]
\[ \sigma^2 = npq \]
\[ \sigma = \sqrt{npq} \]

Ex 5). A coin is tossed 4 times. Find the mean, variance, and standard deviation of the number of heads that will be obtained.

\[ \mu = 4 \left( \frac{1}{2} \right) = 2 \text{ heads} \]
\[ \sigma^2 = 4 \left( \frac{1}{2} \right) \left( \frac{1}{2} \right) = 1 \]
\[ \sigma = \sqrt{1} = 1 \]
Ex 6). A die is rolled 480 times. Find the mean, variance, and standard deviation of the number of 3s that will be rolled.

\[ p = \text{# of 3s} \]
\[ q = \text{everything else} \]
\[ n = 480 \] \( \left( \frac{1}{6} \right) = 80 \] 3s
\[ \mu = 480 \left( \frac{1}{6} \right) = 80 \] 3s
\[ \sigma^2 = 480 \left( \frac{1}{6} \right) \left( \frac{5}{6} \right) = 66.6666666667 \]
\[ \sigma = \sqrt{66.6666666667} = 8.16 \]

Ex 7). The Statistical Bulletin published by Metropolitan Life insurance Co. reported that 2% of all American births result in twins. If a random sample of 8000 births is taken, find the mean, variance, and standard deviation of the number of births that would result in twins.

\[ \mu = 8000 \times (0.02) = 160 \text{ twins} \]
\[ \sigma^2 = 8000 \times (0.02) \times (0.98) = 156.8 \]
\[ \sigma = \sqrt{156.8} = 12.5 \]
Homework:
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