

6-1

Reteaching**Solving Systems by Graphing**

Graphing is useful for solving a system of equations. Graph both equations and look for a point of intersection, which is the solution of that system. If there is no point of intersection, there is no solution.

Problem

What is the solution to the system? Solve by graphing. Check.

$$x + y = 4$$

$$2x - y = 2$$

Solution

$$y = -x + 4$$

$$y = 2x - 2$$

$$y = -x + 4$$

$$0 = -x + 4$$

$$x = 4$$

$$y = 2x - 2$$

$$0 = 2(x) - 2$$

$$2 = 2x, x = 1$$

Put both equations into y -intercept form, $y = mx + b$.

The first equation has a y -intercept of $(0, 4)$.

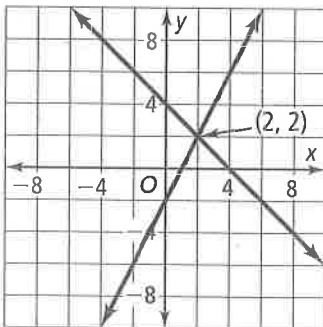
Find a second point by substituting in 0 for y and solve for x .

You have a second point $(4, 0)$, which is the x -intercept.

The second equation has a y -intercept of $(0, -2)$.

Find a second point by substituting in 0 for y and solve for x .

You have a second point for the second line, $(1, 0)$.



Plot both sets of points and draw both lines. The lines appear to intersect $(2, 2)$, so $(2, 2)$ is the solution.

Check

If you substitute in the point $(2, 2)$, for x and y in your original equations, you can double-check your answer.

$$x + y = 4 \quad 2 + 2 \stackrel{?}{=} 4, \quad 4 = 4 \checkmark$$

$$2x - y = 2 \quad 2(2) - 2 \stackrel{?}{=} 2, \quad 2 = 2 \checkmark$$

6-1

Reteaching (continued)**Solving Systems by Graphing**

If the equations represent the same line, there is an infinite number of solutions, the coordinates of any of the points on the line.

Problem

What is the solution to the system? Solve by graphing. Check.

$$2x - 3y = 6$$

$$4x - 6y = 18$$

Solution

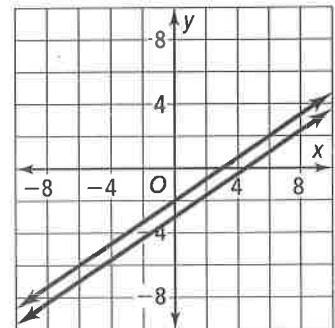
What do you notice about these equations? Using the y -intercepts and solving for the x -intercepts, graph both lines using both sets of points.

$$y = \frac{2}{3}x - 2$$

$$y = \frac{2}{3}x - 3$$

Graph equation 1 by finding two points: $(0, -2)$ and $(3, 0)$. Graph equation 2 by finding two points $(0, -3)$ and $(4.5, 0)$.

Is there a solution? Do the lines ever intersect? Lines with the same slope are parallel. Therefore, there is no solution to this system of equations.

**Exercises**

Solve each system of equations by graphing. Check.

$$\begin{aligned} 1. \quad & 2x = 2 - 9y \\ & 21y = 4 - 6x \end{aligned}$$

$$\begin{aligned} 2. \quad & 2x = 3 - y \\ & y = 4x - 12 \end{aligned}$$

$$\begin{aligned} 3. \quad & y = 1.5x + 4 \\ & 0.5x + y = -2 \end{aligned}$$

$$\begin{aligned} 4. \quad & 6y = 2x - 14 \\ & x - 7 = 3y \end{aligned}$$

$$\begin{aligned} 5. \quad & 3y = -6x - 3 \\ & y = 2x - 1 \end{aligned}$$

$$\begin{aligned} 6. \quad & 2x = 3y - 12 \\ & \frac{1}{3}x = 4y + 5 \end{aligned}$$

$$\begin{aligned} 7. \quad & 2x + 3y = 11 \\ & x - y = -7 \end{aligned}$$

$$\begin{aligned} 8. \quad & 3y = 3x - 6 \\ & y = x - 2 \end{aligned}$$

$$\begin{aligned} 9. \quad & y = \frac{1}{2}x + 9 \\ & 2y - x = 1 \end{aligned}$$

6-2

Reteaching**Solving Systems Using Substitution**

You can solve a system of equations by substituting an equivalent expression for one variable.

Problem

Solve and check the following system:

$$x + 2y = 4$$

$$2x - y = 3$$

Solution $x + 2y = 4$

$$x = 4 - 2y$$

$$2(4 - 2y) - y = 3$$

$$8 - 4y - y = 3$$

$$8 - 5y = 3$$

$$8 - 8 - 5y = 3 - 8$$

$$-5y = -5$$

$$y = 1$$

$$x + 2(1) = 4$$

$$x + 2 - 2 = 4 - 2$$

$$x = 2$$

The first equation is easiest to solve in terms of one variable.

Get x to one side by subtracting $2y$.

Substitute $4 - 2y$ for x in the second equation.

Distribute.

Simplify.

Subtract 8 from both sides.

Divide both sides by -5 .

You have the solution for y . Solve for x .

Substitute in 1 for y in the first equation.

Subtract 2 from both sides.

The solution is $(2, 1)$.

Check Substitute your solution into either of the given linear equations.

$$x + 2y = 4$$

$$2 + 2(1) \stackrel{?}{=} 4$$

$$4 = 4 \checkmark$$

Substitute $(2, 1)$ into the first equation.

You check the second equation.

Exercises

Solve each system using substitution. Check your answer.

1. $x + y = 3$
 $2x - y = 0$

2. $x - 3y = -14$
 $x - y = -2$

3. $2x - 2y = 10$
 $x - y = 5$

4. $4x + y = 8$
 $x + 2y = 5$

6-2

Reteaching (continued)

Solving Systems Using Substitution

Problem

Solve and check the following system:

$$\frac{x}{2} - 3y = 10$$

$$3x + 4y = -6$$

$$\begin{aligned} \text{Solve} \quad \frac{x}{2} - 3y &= 10 \\ \frac{x}{2} &= 10 + 3y \\ x &= 20 + 6y \end{aligned}$$

$$3x + 4y = -6$$

$$3(20 + 6y) + 4y = -6$$

$$60 + 22y = -6$$

$$22y = -66, y = -3$$

$$\frac{x}{2} - 3(-3) = 10$$

$$\frac{x}{2} + 9 = 10$$

$$x = 2$$

First, isolate x in the first equation.Add $3y$ to both sides and simplify.

Multiply by 2 on both sides.

Substitute $20 + 6y$ for x in second equation.

Simplify.

Subtract 60 from both sides.

Divide by 22 to solve for y .Substitute -3 in the first equation.

Simplify.

Solve for x .The solution is $(2, -3)$.

$$\begin{aligned} \text{Check} \quad 3(2) + 4(-3) &\stackrel{?}{=} -6 \\ -6 &= -6 \checkmark \end{aligned}$$

Now you check the first equation.

Exercises

Solve each system using substitution. Check your answer.

$$\begin{aligned} 5. \quad -2x + y &= 8 \\ 3x + y &= -2 \end{aligned}$$

$$\begin{aligned} 6. \quad 3x - 4y &= 8 \\ 2x + y &= 9 \end{aligned}$$

$$\begin{aligned} 7. \quad 3x + 2y &= 25 \\ 2x + 3y &= -6 \end{aligned}$$

$$\begin{aligned} 8. \quad 6x - 5y &= 3 \\ x - 9y &= 25 \end{aligned}$$

6-3 Reteaching

Solving Systems Using Elimination

Elimination is one way to solve a system of equations. Think about what the word “eliminate” means. You can eliminate either variable, whichever is easiest.

Problem

Solve and check the following system of linear equations.

$$4x - 3y = -4$$

$$2x + 3y = 34$$

Solution The equations are already arranged so that like terms are in columns.

Notice how the coefficients of the y -variables have the opposite sign and the same value.

$$4x - 3y = -4$$

$$2x + 3y = 34$$

$$\hline 6x = 30$$

$$x = 5$$

$$4(5) - 3y = -4$$

$$20 - 3y = -4$$

$$-3y = -24$$

$$y = 8$$

Add the equations to eliminate y .

Divide both sides by 6 to solve for x .

Substitute 5 for x in one of the original equations and solve for y .

The solution is $(5, 8)$.

Check

$$4x - 3y = -4$$

$$4(5) - 3(8) \stackrel{?}{=} -4$$

$$20 - 24 \stackrel{?}{=} -4$$

$$-4 = -4 \checkmark$$

Substitute your solution into both of the original equations to check.

You can check the other equation.

Exercises

Solve and check each system.

1. $3x + y = 3$

$$-3x + y = 3$$

2. $6x - 3y = -14$

$$6x - y = -2$$

3. $3x - 2y = 10$

$$x - 2y = 6$$

4. $4x + y = 8$

$$x + y = 5$$

6-3

Reteaching (continued)

Solving Systems Using Elimination

If none of the variables has the same coefficient, you have to multiply before you eliminate.

Problem

Solve the following system of linear equations. $-2x - 3y = -1$
 $5x + 4y = 6$

Solution

$$5(-2x - 3y) = (-1)5$$

$$2(5x + 4y) = (6)2$$

$$-10x - 15y = -5$$

$$\begin{array}{r} 10x + 8y = 12 \\ -7y = 7 \end{array}$$

$$y = -1$$

$$5x + 4(-1) = 6$$

$$5x - 4 = 6$$

$$5x = 10$$

$$x = 2$$

The solution is $(2, -1)$.

Check $-2x - 3y = -1$

$$-2(2) - 3(-1) \stackrel{?}{=} -1$$

$$-1 = -1 \checkmark$$

Multiply the first equation by 5 (all terms, both sides) and the second equation by 2. You can eliminate the x variable when you add the equations together.

Distribute, simplify and add.

Divide both sides by 7.

Substitute -1 in for y in the second equation to find the value of x .

Simplify.

Add 4 to both sides.

Divide by 5 to solve for x .

Substitute your solution into both original equations.

You can check the other equation.

Exercises

Solve and check each system.

5. $x - 3y = -3$

$$-2x + 7y = 10$$

7. $3x + 10y = 5$

$$7x + 20y = 11$$

6. $-2x - 6y = 0$

$$3x + 11y = 4$$

8. $4x + y = 8$

$$x + y = 5$$